# *The pattern of the triangle T(n,k) developing the power function n^9. Description of the method of finding triangles based on the case for the function f(n)=n ^ 5.*

**I**. In my previous paper ( https://drive.google.com/open?id=1DfP4ED3sNsQlzaNEHoXRxWhr-b3lkPxK ) I presented patterns of triangles that develop power functions ***n^5*** and ***n^7***. That document was created as an answer for the question asked by Petro Kolosov in his ealier paper ( https://kolosovpetro.github.io/pdf/Overview_of_preprint_1603.02468.pdf ). Recently I have found a pattern of triangle which develop the function *f(n)= n^m* for power *m*=9. Below it is presented this pattern:

$$T(n,k)=630*k^4*(n-k)^4-120*k*(n-k)+1 \qquad (1)$$

For the triangle *(1):*

$$\sum_{k=1}^{n} T(n,k)=n^9$$

for each *n* natural. In addition *T(n,0)=T(n,n)*=1.

As in previous cases, there is exist a linear relationship beetween neighboring rows of the traingle *(1)*:

$$5*T(n,k)=5*T(n+3,k)-T(n+4,k)-10*T(n+2,k)+10*T(n+1,k)+T(n-1,k)$$

**II**. As I mentioned in my previous paper in order to get a pattern of a triangle one should solve a system of equations where unknows are polynomial coefficients and a variable of the polynom is *k(n-k)* . As example of the application of this method let's try to find the triangle for the ***n^5*** function.

We suspect that a triangle should have the following form:

$$T(n,k)=Ak^2(n-k)^2+Bk(n-k)+C$$

where *A, B, C* are unknown coefficients .

Of course:

$$\sum_{k=1}^{n}\left(Ak^2(n-k)^2+Bk(n-k)+C\right)=n^5 \quad \text{for each n natural,} \qquad (2)$$

By converting the left side of the equation *(2)* we get:

$$A\sum_{k=1}^{n}k^2(n-k)^2+B\sum_{k=1}^{n}k(n-k)+Cn=$$

$$A\sum_{k=1}^{n}k^2\left(n^2-2nk+k^2\right)+B\sum_{k=1}^{n}\left(kn-k^2\right)+Cn=$$

$$A\sum_{k=1}^{n}\left(k^2n^2-2nk^3+k^4\right)+B\sum_{k=1}^{n}\left(kn-k^2\right)+Cn=$$

$$An^2\sum_{k=1}^{n}k^2-2An\sum_{k=1}^{n}k^3+A\sum_{k=1}^{n}k^4+Bn\sum_{k=1}^{n}k-B\sum_{k=1}^{n}k^2+Cn \qquad (3)$$

Thus, we have received an expression containing sums of powers of succesive natural numbers, where powers are 1,2,3,4. In discrete mathematics there are formulas allowing to calculate such sums. These formulas contain so-called Bernoulli numbers. For mentioned powers of succesive natural numbers formulas are following:

$$\sum_{k=1}^{n}k=\frac{n^2+n}{2}$$

$$\sum_{k=1}^{n}k^2=\frac{2n^3+3n^2+n}{6}$$

$$\sum_{k=1}^{n}k^3=\frac{n^4+2n^3+n^2}{4}$$

$$\sum_{k=1}^{n}k^4=\frac{6n^5+15n^4+10n^3-n}{30}$$

If we put in above patterns to the expression *(3),* than we get

$$An^2\left[\frac{2n^3+3n^2+n}{6}\right]-2An\left(\frac{n^4+2n^3+n^2}{4}\right)+A\left(\frac{6n^5+15n^4+10n^3-n}{30}\right)+Bn\left(\frac{n^2+n}{2}\right)-B\left(\frac{2n^3+3n^2+n}{6}\right)+Cn$$

After several standard transformations of the last expression we get

$$\frac{An^5-An+30Cn}{30}+B\left(\frac{n^3-n}{6}\right) \qquad (4)$$

We shall remember that the expression *(4)* is the left side of the input equation *(2)* . So that we have

$$\frac{An^5 - An + 30Cn}{30} + B\left(\frac{n^3 - n}{6}\right) = n^5 \qquad (5)$$

In order to satisfy *(5)* for each *n* natural, coefficients *A,B,C* should be a solution of following system of 3 equations:

$$\begin{cases} \frac{A}{30} & = & 1 \\ B & = & 0 \\ 30 * C - A & = & 0 \end{cases}$$

The only solution of the above system is : *A=30, B=0, C=1*. In the final result the pattern of triangle developing the function ***n^5*** is:

$T(n,k) = 30 * k^2 * (n-k)^2 + 1$.

*Thanks for reading,*
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