The problem of the triangle T(n,k) developing the polynomial function f(n)= n^m. Solution of cases for m=5 and m= 7.

I. Petro Kolosov in his paper https://kolosovpetro.github.io/pdf/Overview_of_preprint_1603.02468.pdf presented the triangle *T(n,k),* which develops the polynomial function ***f(n)=n^3*** for each n - natural number. In that triangle where *n* is row number and *k* is column number, sum of *n* – row beyond 0-item (i.e. T(0,n)) is always equal to *n^3*. The pattern of that triangle is as follows:

$$T(n,k) = 6 * k * (n-k) + 1 \qquad (1)$$

In mathematics, we can find examples of similar triangles, e.g. the Pascal's triangle $T(n,k) = \binom{n}{k}$,

where sum of each complete n-row is equal to 2^n .

Triangle *(1)* is symmetrical e.i. *T(n,k) = T(n,n-k).* Additionally, for each row the first and last item is equal to 1. Beyond that, there is a linear dependence beetween neighboring rows:

$$2 * T(n,k) = T(n+1,k) + T(n-1,k)$$

In paper https://kolosovpetro.github.io/pdf/series_representation_of_power_function.pdf Petro Kolosov used the triangle *(1)* in a development of the polynomial function *n ^ m* where power *m* is greater than 3 and also he presented the exponential function in a different non-standard form. In his works, he posed the question of whether there are similar triangles like *(1)* that are developments in polynomial functions with powers greater than 3 ? At the same time, a given triangle should be symmetrical and have similar properties, e.g. the first and last item of each row should be fixed (e.g. equal to 1).

II. Below is a triangle pattern which is an extension of the ***n ^ 5*** function:

$$T(n,k) = 30 * k^2 * (n-k)^2 + 1 \qquad (2)$$

For the triangle *(2):*

$$\sum_{k=1}^{n} T(n,k) = n^5$$

for each *n* natural. Additionally of course *T(n,0)=T(n,n)*=1.

There is also here a linear relationship between neighboring rows:

$$3*T(n,k)=3*T(n+1,k)-T(n+2,k)+T(n-1,k)$$

III. The case of a triangle for the polynomial function ***n^7*** we can present as follows:

$$T(n,k)=140*k^3*(n-k)^3-14*k*(n-k)+1 \qquad (3)$$

For the triangle *(3):*

$$\sum_{k=1}^{n} T(n,k)=n^7$$

for each *n* natural. In addition *T(n,0)=T(n,n)*=1.

As in previous cases, we can present also a linear relationship for *(3)*:

$$4*T(n,k)=T(n+3,k)-4*T(n+2,k)+6*T(n+1,k)+T(n-1,k)$$

IV. General describtion ot the method getting the triangles.
In order to get patterns *(2)* and *(3)* one should solve a system of equations where unknows are polynomial coefficients and a variable of the polynom is *k*(n-k*) . For example in the case of the triangle for the n^7 function there are obtained coefficients : 140, 0, -14, 1.

V. Conclusions.
Above solutions show that the answer on the question asked is yes, but I have not found similiar patterns of triangles for power functions, where power is an even number, e.g. 4 or 6. It can be assumed that for each odd number *m* there is a similiar triangle, that develops the polynomial function *f(n)=n^m*. At the same time I am not sure it, because it is only hipothese, which should (can) be proven.

Thanks for reading,
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